Airtightness of Buildings – Considerations regarding the Zero-Flow Pressure and the Least Square Regression

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ABSTRACT

This paper discusses two particular points of the buildings airtightness measurement method (ISO 9972) in relation with the calculation of the combined standard uncertainty: (1) the zero-flow pressure difference and (2) the line of organic correlation.

The zero-flow pressure difference is measured at the start and the end of the test in order to calculate the change of pressure caused by the fan or blower door. Actually the zero-flow pressure difference fluctuates during the test in function of the wind and the temperature difference between inside and outside the building. One should therefore take this fluctuation into account in the uncertainty of the induced pressure difference. The paper shows how it could be done.

The air flow coefficient and air flow exponent are generally determined using an ordinary least squares regression technique (OLS). This is however not the most appropriate technique because there are uncertainties in both the measured air flow rates and the pressure differences. The paper shows how the line of organic correlation (LOC) could be used in order to take these uncertainties into account.

INTRODUCTION

In European countries, increasing importance has been given to airtightness of buildings since the first publication of the directive on the energy performance of buildings in 2002. In some countries there even are requirements or financial incentives linked with the airtightness level. It is therefore more and more important to pay attention to the uncertainty of airtightness measurements.

The issue of uncertainty of airtightness measurements has already been dealt with in various publications (Persily 1985, Sherman 1994, Delmotte 2013, Walker 2013) but is still incompletely solved in practice. This was also a point of discussion during the last revision of ISO 9972.

This paper discusses two particular points of the buildings airtightness measurement method in relation with the calculation of the combined standard uncertainty: (1) the zero-flow pressure difference and (2) the line of organic correlation.

Theoretical developments are translated into practical formulae which could be applied in daily practice.

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ZERO-FLOW PRESSURE DIFFERENCE

Pressure difference induced by the fan

In given climatic conditions (wind and temperature) and in the absence of fan, pressure differences $\Delta p_{0,j}$ are naturally generated across the envelope of the building. The equilibrium internal pressure is such that the airflow that enters the building is equal to the flow that leaves. The sum of the airflows through the building envelope is therefore equal to zero (formally we should talk about mass flow). Accordingly, parts of the envelope must necessarily undergo underpressure while others are in overpressure.

In the absence of wind or temperature difference, the action of a fan located in the building envelope induces an identical pressure difference Δp across all points of the envelope. However, this is not quite true because the internal partitioning of the building may generate pressure losses. ISO 9972 requires opening all interior doors in order to minimize this effect.

When adding the effect of a fan to that of the wind and of the temperature difference, each point (j) of the envelope is subjected to a pressure difference $\Delta p_{m,j}$ equal to the sum of those it would undergo for each of the two separate effects (Δp and $\Delta p_{0,j}$) (Sherman 1990) (Formula 1 and Figure 1). Each point thus undergoes a similar change in pressure while keeping its relative difference compared to the other points. Note that this principle of addition is not true for air flow rates.

$$\Delta p_{m,j} = \Delta p_{0,j} + \Delta p \qquad (1)$$

Additivity of pressure differences is used in ISO 9972 to indirectly measure the pressure difference induced by the fan:

1. Zero-flow pressure difference Δp_0 - Pressure difference is measured at one point of the envelope when the building is subject to natural conditions only (fan off and covered)
2. Measured pressure difference Δp_m - Pressure difference is measured at the same point of the envelope when the fan is operating
3. Induced pressure difference Δp - Pressure difference induced by the fan is calculated by subtracting the first value from the second (Formula 2)

$$\Delta p = \Delta p_m - \Delta p_0 \qquad (2)$$

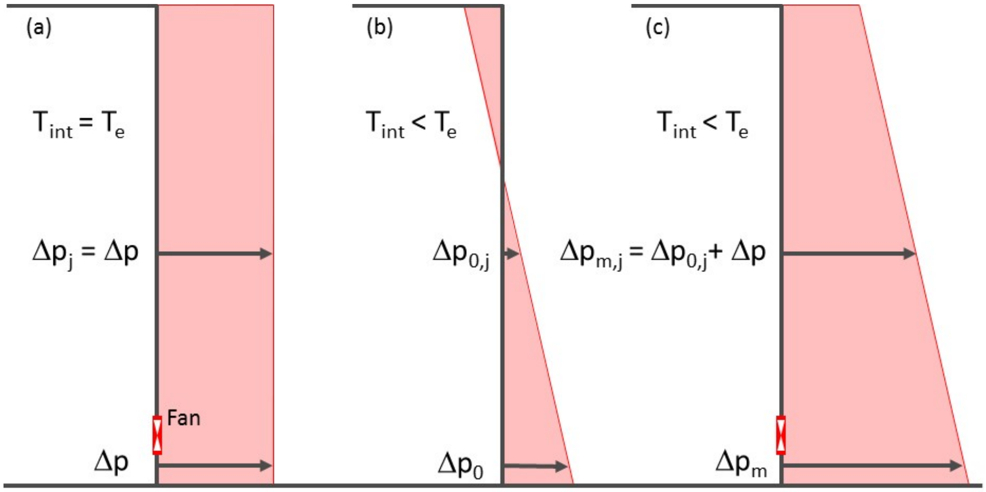

Figure 1 Example of pressure distribution over the height of a building (a) for a fan only, (b) for a temperature difference only and (c) for the combination of the fan and the temperature difference.

Complete Building Pressurization

The calculation model adopted by ISO 9972 assumes the entire building envelope is either pressurized or depressurized. It is therefore necessary that the pressure difference induced by the fan overcomes the pressure differences generated by climatic conditions. In this way for example, a natural depression of -3 Pa (-0,06 lb/ft²) could be overcome by an induced pressure of 8 Pa (0,17 lb/ft²).

The fact that ISO 9972 requires the zero-flow pressure difference to be less than 5 Pa (0,10 lb/ft²) in absolute value and the lowest measured pressure difference (Δp_m) to be at least 5 times the zero-flow pressure difference with a minimum of 10 Pa (0,21 lb/ft²) aims to respect the calculation model.

Although it is not specified in ISO 9972, this implies that the zero-flow pressure difference is measured at a point where the pressure difference generated by the climatic conditions is a priori the largest. In practice, the measuring point is usually located close to the blower door (itself often installed on the ground floor of the building) and there is no guarantee that this is the best location.

Average pressure difference on the envelope

The calculation model adopted by ISO 9972 also assumes that the pressure difference between inside and outside the building is identical at all points of the envelope. This is only possible in the absence of wind and temperature difference and is therefore virtually impossible to satisfy in practice.

To overcome this problem, ISO 9972 requires the pressure difference induced by the fan to be much greater than the absolute value of the pressure differences generated by climatic conditions. In this way, variations in the pressure difference remain relatively low and it is assumed that the hypothesis of the model are fulfilled. Under these conditions ISO 9972 assumes the average value of the pressure difference which is applied to the envelope to be equal to the pressure induced by the fan. This is an approximation because the average value of the zero-flow pressure difference is not necessarily equal to zero (even if the total airflow is equal to zero).

When the fan puts the whole building envelope in positive or negative pressure, the airflow through the fan is equal to the air flow rate through the envelope. Following the model of ISO 9972, this flow is associated with the pressure induced by the fan and eventually allows to characterize the air permeability of the envelope.

Measurement uncertainty

When measuring the airtightness of a building, it is not possible to measure the zero-flow pressure during the test and the climatic conditions most generally don't remain constant (especially due to wind that can quickly change intensity and direction). So ISO 9972 requires the zero-flow pressure difference Δp_0 to be measured for at least 30 seconds both at the start (Δp_{01}) and the end (Δp_{02}) of the test. However, nothing prevents climatic conditions being different during the test. In addition to the uncertainty of the measures themselves, some variability in climatic conditions should also be taken into consideration.

A typical example of evolution of the zero-flow pressure difference at the start (30 seconds), during (10 minutes) and at the end (30 seconds) of a test is shown in Figure 2. In order to find the most probable value of this pressure during the test, ISO 9972 takes the mean of Δp_{01} and Δp_{02}.

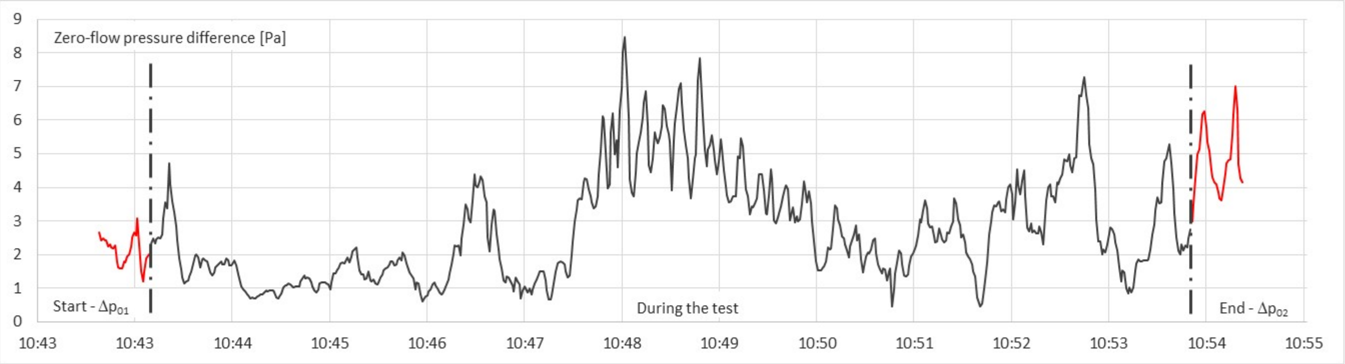

Figure 2 Example of evolution of the zero-flow pressure difference at the start (30seconds – max: 3,1 Pa – mean: 2,1 Pa – min: 1,2 Pa) , during (10 minutes – max: 8,5 Pa – mean: 2,7 Pa – min: 0,5 Pa) and at the end (30seconds – max: 7,0 Pa – mean: 4,8 Pa – min: 3,0 Pa) of a test.

The zero-flow pressure difference is no constant value. So one cannot calculate the Type A standard uncertainty (JCGM 2008) which would require to carry out several measurements of the same value. One should therefore calculate the Type B standard uncertainty (scientific judgement based on all of the available information on the possible variability).

In order to take some variability in climatic conditions into account, one could consider the minimum and maximum Δp_0 values measured at the start and the end of the test. This should be added to the combined standard uncertainty of the induced pressure difference (Formula 4).

$$\Delta p = \Delta p_m - \frac{\Delta p_{0,1} + \Delta p_{0,2}}{2} \tag{3}$$

$$u_c(\Delta p) = \sqrt{u^2(\Delta p_m) + \frac{u^2(\Delta p_{0,1})}{4} + \frac{u^2(\Delta p_{0,2})}{4} + \left(\frac{max(|\Delta p_{0,max} - \Delta p_{0,av}|;|\Delta p_{0,min} - \Delta p_{0,av}|)}{\sqrt{6}}\right)^2} \tag{4}$$

In order to reduce the uncertainty related to the modelling, it could be worth measuring the pressure difference at several points of the envelope. This would give more chances to catch the greatest zero-flow pressure difference and determine the lowest measurement stage accordingly. In this sense, looking for the location of the measurement point that provides the lowest zero-flow pressure difference in order to comply with the criteria of ISO 9972 is not recommended.

LINE OF ORGANIC CORRELATION

Applicability of least squares

In the framework of the buildings airtightness measurement, ISO 9972 assumes the relation between the airflow rate and the pressure difference has an exponential form (Formula 5).

$$q_{env} = C_{env} \cdot \Delta p^{n} \quad (5)$$

This exponential relation can be transformed into a linear relation as follows:

$$\ln q_{env} = \mathrm{n} \cdot \ln \Delta p + \ln C_{env} \quad (6)$$

ISO 9972 requires the use of a least squares technique for the calculation of the airflow coefficient C_{env} and the airflow exponent n based on the measurement points (Δp_i , $q_{env,i}$) but does not give further guidance.
The Ordinary method of Least Squares (OLS) is applicable when all y_i values ($y_i = \ln q_{env,i}$) are equally uncertain ($u_c(y_1)= u_c(y_2)=\cdots=u_c(y_n)$) and the uncertainties on x_i values ($x_i = \ln \Delta p_i$) are negligible (Delmotte 2013). When uncertainties of y_i values are not equal (and uncertainties of x_i values are negligible), it is advisable to use the Weighted method of Least Squares (WLS).
None of these two methods are theoretically applicable to the buildings airtightness measurement because both sets of x_i and y_i values have non negligible and unequal uncertainties. It is therefore proposed to examine the possibility of using the Weighted Line of Organic Correlation (WLOC) which takes both sets of uncertainties into account.

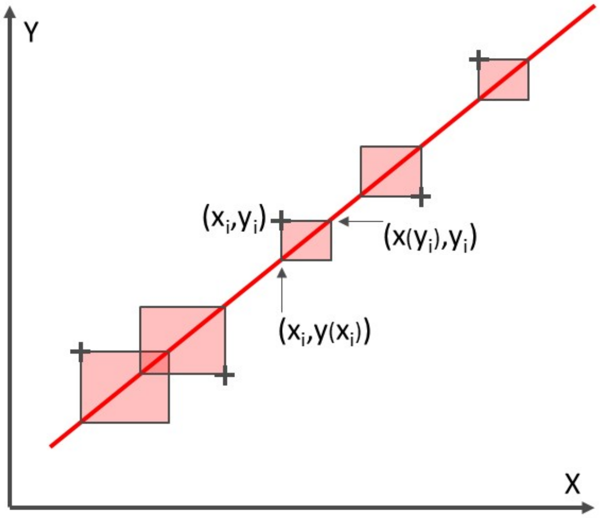

Figure 3 The weighted line of organic correlation minimalizes the sum of the products of the weighted vertical and horizontal differences between the measurement points and the line.

Description of the method

The WLOC consists of finding the regression line y = a x + b that minimalizes the sum of the products of the weighted vertical and horizontal differences between the measurement points and the line (Figure 3); which comes to minimalizing the following sum:

$$\sum_{i=1}^{N}\big(w_{xi}|x_i - x(y_i)| \cdot w_{yi}|y_i - y(x_i)|\big) \tag{7}$$

$$\sum_{i=1}^{N}\big(w_{xi}|x_i - (y_i - b)/a| \cdot w_{yi}|y_i - (a\,x_i + b)|\big) \tag{8}$$

The weights w_{ix} and w_{iy} applied to each measurement point i are equal to:

$$w_{ix} = \frac{1}{s^2(x_i)} = \frac{1}{u_c^2(x_i)} \tag{9}$$

$$w_{iy} = \frac{1}{s^2(y_i)} = \frac{1}{u_c^2(y_i)} \tag{10}$$

Constants a and b of this regression line are calculated as follows:
(For the sake of simplification $\sum x_i$ is used for $\sum_{i=1}^{N} x_i$)

$$a = \frac{\sqrt{\sum w_{xi}\,w_{yi} \sum w_{xi}\,w_{yi}\,y_i^2 - \left(\sum w_{xi}\,w_{yi}\,y_i\right)^2}}{\sqrt{\sum w_{xi}\,w_{yi} \sum w_{xi}\,w_{yi}\,x_i^2 - \left(\sum w_{xi}\,w_{yi}\,x_i\right)^2}} = \frac{S_y}{S_x} \tag{11}$$

$$b = \frac{\sum w_{xi}\,w_{yi}\,y_i - a \sum w_{xi}\,w_{yi}\,x_i}{\sum w_{xi}\,w_{yi}} = \bar{y} - \frac{S_y}{S_x}\,\bar{x} \tag{12}$$

a and b are eventually used for the calculation of n and C_{env} :

$$n = a \tag{13}$$

$$C_{env} = e^{b} \tag{14}$$

Other important characteristics of the weighted line of organic correlation are the following:

Weighted mean

$$\bar{x} = \frac{\sum w_{xi}\,w_{yi}\,x_i}{\sum w_{xi}\,w_{yi}} \tag{15}$$

$$\bar{y} = \frac{\sum w_{xi}\,w_{yi}\,y_i}{\sum w_{xi}\,w_{yi}} \tag{16}$$

Variance

$$S_x^2 = \frac{\sum w_{xi}\,w_{yi}(x_i - \bar{x})^2}{\sum w_{xi}\,w_{yi}} \tag{17}$$

$$S_y^2 = \frac{\sum w_{xi}\,w_{yi}(y_i - \bar{y})^2}{\sum w_{xi}\,w_{yi}} \tag{18}$$

Covariance

$$S_{xy} = \frac{\sum w_{xi}\,w_{yi}(x_i - \bar{x})\,(y_i - \bar{y})}{\sum w_{xi}\,w_{yi}} \tag{19}$$

Coefficient of determination

$$r^2 = \frac{S_{xy}^2}{S_x^2\,S_y^2} \tag{20}$$

Advantages of WLOC is that it minimizes errors in both X and Y directions and that it provides a unique line identical regardless of which variable, X or Y, is used as the response variable (Helsel and Hirsch 2002) (Figure 4). Considering that the regression lines pass through the centroid of the data $(\bar{x}, \bar{y})$, this property becomes more important as one is interested in the estimate of the air leakage rate at low pressure difference (e.g. 4 or 10 Pa) (0,08 or 0,21 lb/ft²). Applying both OLS and WLOC on a sample of 20 buildings airtightness measurements from about 10 to 100 Pa (0,21 to 2,1 lb/ft²) has shown differences up to 8% for q_4 and 6% for q_{10} while they were limited to 2% for q_{50} (Figure 5).

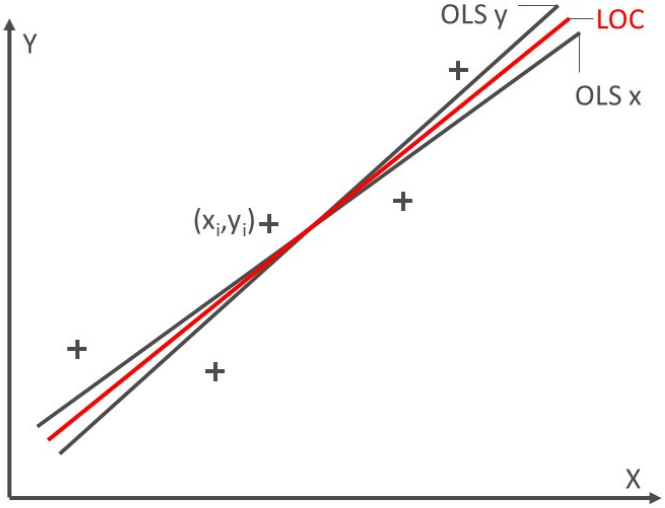

Figure 4 The weighted line of organic correlation provides a unique line identical regardless of which variable, X or Y, is used as the response variable.

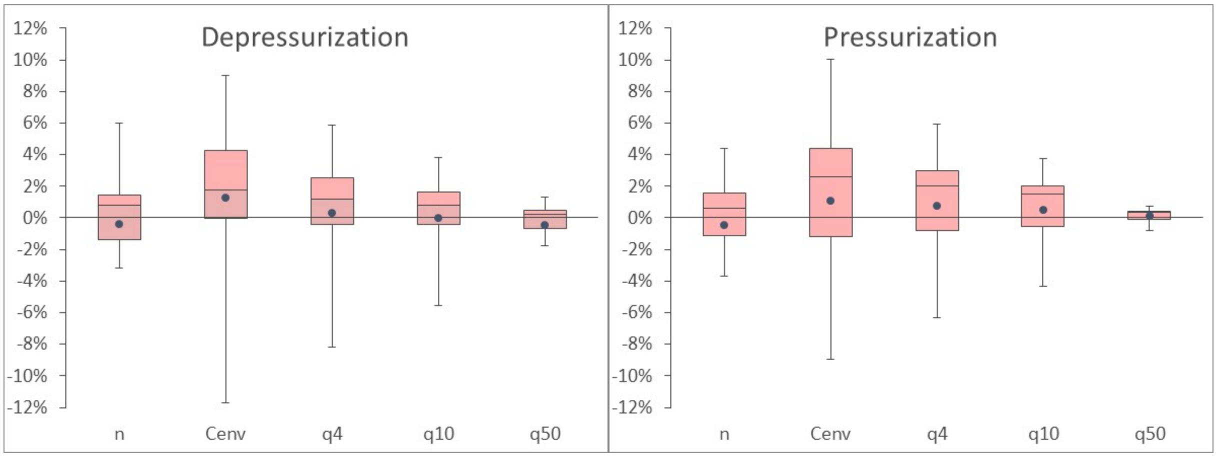

Figure 5 The difference between WLOC and OLS can be relatively high as the reference pressure goes away from the centroid of the data (about 50 Pa in this case) – Sample of 20 buildings airtightness measurements from about 10 to 100 Pa.

Further calculation is needed to propose formulae for the experimental variances of a and b and their estimated correlation coefficient. These values allows to further propagate the uncertainties to the air leakage rate at the reference pressure difference (Delmotte 2013).

NOMENCLATURE

C_{env} = Air flow coefficient
n = Air flow exponent (also refered to as "pressure exponent")
q_{env} = Air flow rate through the building envelope
s(x) = Experimental standard deviation of estimate x
T_{int} = Internal air temperature
T_e = External air temperature
q_4 = Air leakage rate at 4 Pa
$u(\Delta p)$ = Standard uncertainty of estimate Δp
$u_c(x)$ = Combined standard uncertainty of estimate x
w_{xi} = Weight attributed to estimate x_i
Δp = Induced pressure difference
Δp_0 = Zero-flow pressure difference
$\Delta p_{0,av}$ = Average value of $\Delta p_{0,1}$ et $\Delta p_{0,2}$
$\Delta p_{0,max}$ = Maximum value of all Δp_0 values measured at the start and at the end of the test
$\Delta p_{0,min}$ = Minimum value of all Δp_0 values measured at the start and at the end of the test
Δp_m = Measured pressure difference

Subscripts

i = element of a series
j = element of a series
max = *maximum*
min = *minimum*
1 = *start of the test*
2 = *end of the test*

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