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Pile group settlement under vertical static load using pile to pile interaction factor: points of attention and influence of soil nonlinearity

Tassement des groupes de pieux sous chargement vertical à l'aide du facteur d'interaction pieu-pieu: points d'attention et influence de la non-linéarité du sol

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ABSTRACT: Pile group design is mainly a settlement concern since the stiffness of the pile group is generally smaller than the sum of the individual stiffnesses of each pile: stiffness efficiency is less than 1. The pile-to-pile interaction factor (IF) method could be considered as the most rational and physically acceptable method for calculating pile group settlement compared to other semi-empirical methods such as 'equivalent pier' or 'equivalent raft' methods. It superimposes the effects between all possible pairs of piles within a pile group and uses elastic pile-soil-pile interactions. In this paper, old and recently developed closed-form solutions for the IF are reviewed as well as points of attentions that should be considered. It was concluded that the IF approach leads to a huge decrease of the group stiffness even with the consideration of small strain soil stiffness parameters. Furthermore, unlike experimental results, the calculated settlement ratio is independent of the load level. To investigate the effect of the soil nonlinearity on the IF, 3D Finite Element Method (FEM) simulations are performed. It was shown that the IF diminish as pile loading increases. An updated value of the IF in function of the load level provides a better agreement with the measurements.

RÉSUMÉ: Le dimensionnement d'un groupe de pieux est essentiellement un problème de tassement puisque la raideur des pieux sous une semelle rigide est plus faible que la somme des raideurs individuelles de chaque pieu : le facteur de raideur est inférieur à 1. La méthode des facteurs d'interaction pieux/pieux peut être considérée comme la plus cohérente et la plus physiquement acceptable pour le calcul des tassements d'un groupe de pieux en comparant à d'autres méthodes semi-empiriques comme la méthode de 'pieux équivalent' ou la méthode de 'semelle virtuelle'. La méthode superpose les effets entre toutes les paires de pieux possibles au sein du groupe et utilise une interaction pieu-sol-pieu purement élastique. Dans cet article, anciennes et récentes solutions des facteurs d'interactions ont été évaluées ainsi que les points d'attention qui doivent être prises en compte. Il a été conclue que les facteurs d'interaction aboutissent à une forte diminution de la raideur du groupe même en utilisant des paramètres de sol à faible déformations. En outre, contrairement aux résultats expérimentaux, le facteur de tassement calculé est toujours indépendant du niveau de chargement. Pour étudier l'effet de la non linéarité du sol sur les facteurs d'interaction, des simulations à l'aide de la méthode des élément finis (FEM) ont été réalisées. Il a été démontré que les facteurs d'interaction diminuent en fonction du chargement. Une mise à jour des facteurs d'interaction de pieux en fonction du niveau de chargement donne une meilleure correspondance avec les mesures.

KEYWORDS: pile group settlement, interaction factors, FEM, soil nonlinearity.

1 INTRODUCTION.

For the last 60 years, numerous approaches have been proposed for estimating the settlement of pile groups. The principal approaches can be categorized as follows : *(1) the empirical or semi-empirical approach* (Meyerhof 1976); (Vesic 1977), *(2) the equivalent raft or pier approach* (Terzaghi and Peck 1967); (Fellenius 1991); (Poulos 1993); (Yamashita, Tomono, and Kakurai 1987), *(3) the interaction factor (IF) approach* (Poulos 1968); (Poulos and Davis 1980); (Randolph and Wroth 1978); (Poulos 1989) and *(4) the numerical analysis approach* (Ottaviani 1975); (Clancy and Randolph 1996).

The advantages of empirical approaches and the equivalent raft approach is their ease of application for fast design and verification. They lack, however, a physical meaning and generalizing these approaches might be quite difficult. After a proper soil parameters calibration, the numerical approach might be considered as the most rigorous method since it captures all the physical properties, uses less hypotheses and is limited by less restrictions compared to the previous methods. However, it is very time consuming and requires attention and expertise. The IF approach is also a physically accepted approach. The concept was first introduced by (Poulos 1968) where the pile group effect can be assessed by superimposing each time the effect of only two piles. This leads to a decrease in the group stiffness and to a non-uniform distribution of the load in case of rigid cap. The IF was earlier deduced with purely numerical methods, i.e. the boundary element method (Poulos & Davis, 1980). More recently, rather simple and rigorous analytical solutions were developed which made the IF method a very practical and useful tool ((Randolph 2003).

The study of the pile group settlement is considered in this paper where the pile cap is not in contact with the ground. Based on an extensive literature study of IF advances, points of attention and limitations are outlined. Next, 3D finite element method simulations are presented. Special attention is paid to the effect of soil non-linearity on pile-to-pile interaction. Effects of the rigidity of neighboring piles are also studied. Based on the FE results, this paper presents a practical solution to consider the sensitivity of the IF to soil non-linearity. Finally, the proposed method is compared to the results of an experimentally tested pile group.

2 THE PHYSICS AND THE LIMITATIONS OF INTERACTION FACTOR METHODS

2.1 *The physics of the method*

Due to its own load, a vertically loaded pile settles and affects the displacement of neighboring piles. The IF α between two piles is defined as the head settlement of the receiver pile U_{12} divided by the pile head settlement of the loaded 'source' pile U_{11}:

$$\alpha = \frac{U_{12}}{U_{11}} \tag{1}$$

Boundary element and finite element methods were used as first and widespread rigorous numerical techniques to analyze the pile group effect. A much simpler approach using the Winkler method leads to closed-form analytical solutions where good agreement is obtained with results of rigorous solutions ((Randolph 2003). To calculate the IF, the original solution of (Randolph and Wroth 1978) uses a logarithmic decay of the soil shaft displacement with the radial distance s from the pile. The soil attenuation function $\psi(s)$ is based on the plane strain assumption and is expressed as :

$$\psi(s) = \frac{\ln\left(\frac{r_m}{s}\right)}{\ln\left(\frac{2r_m}{d}\right)} \tag{2}$$

where d is the shaft diameter of the 'source' pile and r_m is an empirical radius (so-called 'magical' radius by (Randolph 2003)) beyond which the soil displacement due to the loaded 'source' pile is assumed to be negligible.

r_m can be expressed experimentally or numerically. The following approximation is generally used (Randolph and Wroth 1978):

$$r_m = 2.5\rho L(1 - \nu_s) \tag{3}$$

where L is the pile length, ν_s is the Poisson's ratio of the soil and ρ is an inhomogeneity factor given by the ratio of the average soil shear modulus G_{avg} to the soil shear modulus at the pile base G_L.

(Mylonakis and Gazetas 1998) developed a closed-form solution for the calculation of the IF as defined by (Poulos and Davis 1980) where pile-interaction is not only affected by the loaded 'source' pile but also by the reinforcing effect of neighboring piles (rigidity of the receiver pile and interaction between the pile and the surrounding soil are considered). The IF is expressed as a product between the soil attenuation function $\psi(s)$ and a diffraction factor ζ:

$$\alpha = \psi(s)\zeta \tag{4}$$

where $\zeta = \frac{1}{2}\left[1 - \frac{2\lambda L(\Omega^2-1)+2\Omega}{(\Omega^2+1)\sinh(2\lambda L)+2\Omega\cosh(2\lambda L)}\right]$; λ is the load transfer parameter and Ω is the dimensionless base stiffness.

(Randolph 2003) described this analytical solution as "seminal advance" in qualifying interaction between piles. The general system of the solution of (Mylonakis and Gazetas 1998) consists of "n" vertical compressible piles embedded in an elastic homogeneous soil layer modelled with a Young modulus E and Poisson ratio ν_s. The main advantage of the solution is the possibility to run fast, non-dimensional and parametric analysis for pile group design, in contrast to heavy advanced numerical analyses. For example, Figure 1 shows the evolution of the IF α as a function of the dimensionless parameter s/d for L/d=10 and L/d=100 with E_p/E_s=1000 and ν_s=0.5.

By solving the matrix equation using the IF for a group of n identical piles under a rigid cap, the group settlement U_G can be calculated. To capture the group effect, the settlement ratio $R_s=U_G/U$ (U is the single pile settlement) or the stiffness ratio $K_G/(n*K)$ (K and K_G are respectively the single pile stiffness and the pile group stiffness) are often used. U and K are deduced from closed-form solution using also the Winkler approach (Mylonakis 2001). Figure 2 illustrates an example of the evolution of the settlement ratio R_s for a 3*3 pile group in function of the dimensionless parameter s/d for E_p/E_s=100 and E_p/E_s=1000 with L/d=25 and ν_s=0.5.

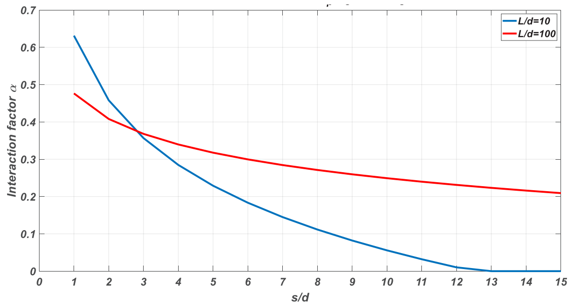


Figure 1. IF as a function of s/d for L/d=10 and L/d=100, E_p/E_s =1000 and υ=0.5

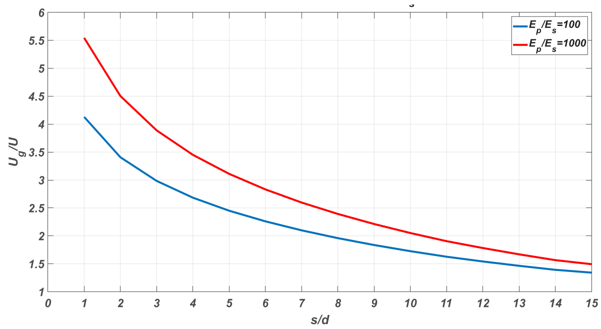


Figure 2. Settlement ratio R_s=Ug/U of a 3*3 pile group for slenderness ratio L/d=25

2.2 *Points of attention*

2.2.1 *Layered and inhomogeneous soils*

Layered and inhomogeneous soil profiles have a great influence on the pile group response. The solution of (Mylonakis and Gazetas 1998) is explicitly given for homogeneous soil. Although the method could be generalized for layered soils, it is less straightforward for more than 2 soil layers. Calculations of the stiffness ratio of a pile group (3*3) in two homogeneous layers are given in Figure 3 to show the importance of soil layering. The first homogeneous layer is located at 1/3 of the pile length with a soil Young modulus E_1. The second layer is the bearing layer with Young modulus E_2. Several ratios of soil stiffnesses E_2/E_1 are considered: E_2/E_1=1 (homogeneous soil), 2 and 4.

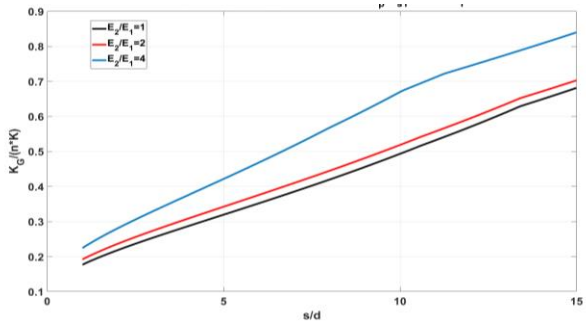


Figure 3. Stiffness ratio $K_G/(n*K)$ for a 3*3 pile group with L/d=20, E_p/E_s=1000 and ν_s=0.4: effects of a layered soil profile.

Even in the same soil formation, soil properties are not homogeneous and soil stiffness generally increases with depth. Assuming an average homogeneous soil is a commonly used assumption in practice. Recently, (Crispin and Leahy 2018) and (Crispin, Leahy, and Mylonakis 2018) developed a closed-form analytical solution based on the Winkler approach using modified Bessel functions for individual piles and pile groups embedded in inhomogeneous soils. A power law function is used to describe the shear modulus variation with depth. To inspect inhomogeneity effect, the solution is implemented in this paper and compared to an equivalent average homogeneous stiffness profile of the soil. Figure 4 shows the settlement ratio R_s of a 2*2 pile group where the shear modulus G is increasing with depth z by G(z)= 4.6 z MPa. The group stiffness in a "inhomogeneous" soil with increasing G with depth is higher (group settlement decreases) than the group stiffness in "homogeneous" soil using an average equivalent stiffness.

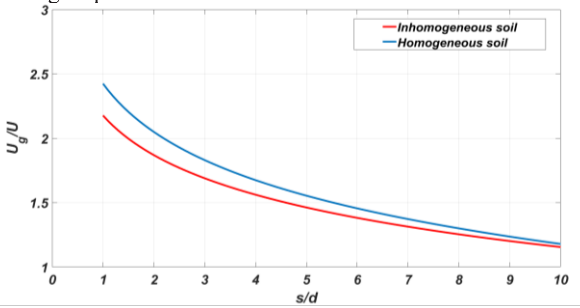


Figure 4. Settlement ratio Rs for 2*2 pile group with L/d=20: homogeneous vs. inhomogeneous soil profile.

2.2.2 *Bearing stratum and underlying layers*

Although IF methods enable a parametric analysis of the soil stiffness at the pile base (constrained base or floating piles), they do not take into account the interaction between piles at the pile base. According to (Randolph and Wroth 1978) and (Mylonakis and Gazetas 1998), the rate of attenuation of the displacement at the pile base is much higher than that at the pile shaft since pile base settlement decays proportionally to the radial distance from the pile. The IF at the pile base is neglected in almost all practical situations. Figure 5 shows the effect of the soil rigidity at the pile base E_b on the settlement ratio for a 3*3 pile group. Even without pile-to-pile interaction at the pile base, the 'shaft' group effect is influenced by the pile base stiffness E_b (interaction decreases as the soil stiffness at the pile base increases).

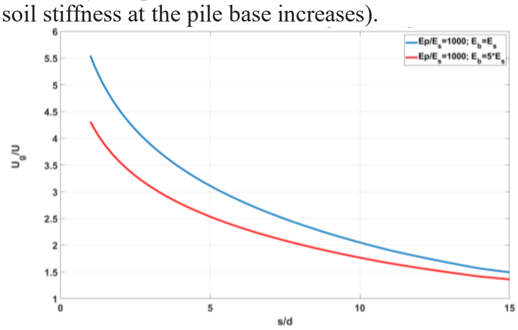


Figure 5. Effect of the soil stiffness at the pile base on the settlement ratio for a 3*3 pile group with L/d=25, and ν_s=0.5

The IF method does not consider the presence of a compressible layer below the pile base. In that case, pile group settlement due to the load transfer to the deep compressible layer should be added to the group settlement obtained from aforementioned IF methods.

2.2.3 *Soil nonlinearity*

(Mandolini and Viggiani 1997) and (Sheil et al. 2019) summarized in their reviews that soil nonlinearity in pile interaction is limited to the pile vicinity while pure elasticity remains between piles. For this reason, most analytical approaches incorporating nonlinear pile group behavior are based on the load transfer of each pile (essentially with hyperbolic transfer functions) coupled with the IF method where elastic soil parameters are used. This means that if nonlinearity is incorporated, only the IF of the pile i (α_{ii}) under its own load is considered (Caputo and Vigiani 1984), (Mandolini and Viggiani 1997):

$$\alpha_{ii} = \frac{1}{1-\frac{Q}{Q_{ult}}}, \quad (5)$$

with Q the applied load and Q_{ult} the ultimate load.

Figure 6 shows the effect of soil nonlinearity according to (Mandolini and Viggiani 1997) on a 2*2 pile group in comparison to a linear problem. It can be observed that the incorporation of nonlinearity for each pile makes the interaction between piles even more unrealistic due to the overestimation of the pile group effect. This exaggerated effect could be explained by the use of elastic soil stiffness in the IF and by the use of elastic individual pile stiffness as reference. To assess the realism of such behavior, the results of a 3D Finite Element model using inelastic soil are discussed in this paper.

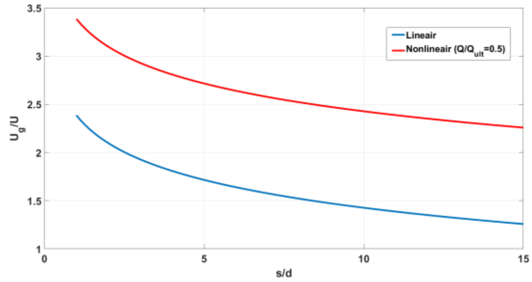


Figure 6. Effect of soil nonlinearity on the settlement ratio for a 2*2 pile group with L/d=25, Ep/Es=1000 and ν_s=0.5.

3 FINITE ELEMENT AND SOIL MODELLING

3.1 *Model description*

The 3D Finite-Element (FE) models are developed in this paper using Plaxis 3D software (Plaxis, 2019). The 3D FE analysis of a large group of piles is a computationally consuming task. To speed up the parametric studies on the IF method, a linear array of piles was considered (Figure 7). A prescribed displacement is imposed to the 'source' pile. A symmetry in the model was taken into account where the free field is placed at sufficiently large distances (Figure 7). A fine mesh was considered in all models with a very fine discretization around the piles. A rigid interface is considered between the pile and the soil. The following pile parameters are considered: pile diameter d=0.5 m, pile length L=10 m, E_p = 25 GPa and ν= 0·2. Three soil models are studied in this research: an elastic model (EL), an elasto-perfectly plastic model with Mohr Coulomb failure criterion MC, and a Hardening soil model (HS model).The soil properties are defined in Table 1 where γ is the soil weight, E the Young modulus, ν_s the Poisson ratio, c the cohesion, φ' the friction angle, ψ the dilatancy angle, E_{50}^{ref} the secant stiffness in drained triaxial test, E_{oed}^{ref} the tangent stiffness in oedometer loading, E_{ur}^{ref} the unloading reloading stiffness and m the Power-low of the stress level dependency on stiffness. In the elastic soil model both constant ("homogeneous" soil) and linearly increasing stiffness ("inhomogeneous" soil with E(z)=5000z) are considered. The average stiffness of the inhomogeneous case corresponds to the considered stiffness in the homogeneous case. Prescribed vertical displacement is first applied on the top of the 'source' pile. The settlement of the ground surface at distances s=3d, 5d and 10d are computed. This represents the IF in the absence of neighboring piles (no reinforcement effect). The same analysis is done with the presence of one neighboring pile each time (only two piles are considered in each analysis) with the same distances (s=3d, 5d and 10d) between 'source' and 'receiver' piles.

3.2 *Numerical Results*

3.2.1 *Numerical analysis vs. analytical solutions:*

Using aforementioned pile and elastic soil parameters, the IF obtained from the FE calculations (FE-EL) are compared to the closed-form solutions (see Table 2). FE calculations and closed-form solutions demonstrate that:

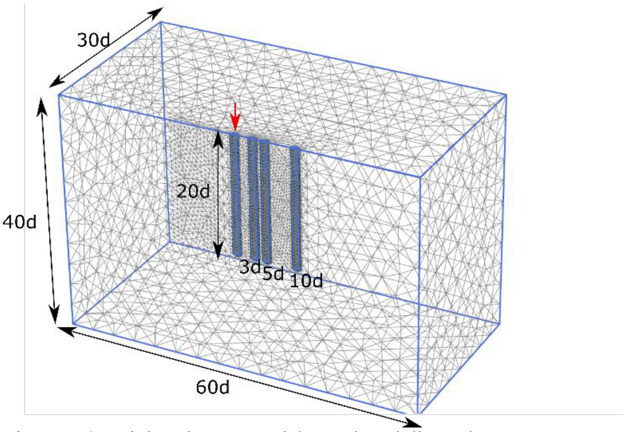


Figure 7. 3D Finite Element model : mesh and dimensions

- The IF decreases with the radial distance from the pile. In elastic analyses non negligible interactions between piles can be observed even beyond a radial distance of 10d.
- The reinforcement effect slightly decreases the IF between piles.
- Equivalent average homogeneous stiffness in the case of inhomogeneous soil overestimates pile interactions compared to the model with increasing stiffness with depth.
- Although a satisfactory tendency is obtained between FE calculations with elastic soils (FE-EL) and closed-form solutions, differences are observed in the absolute values of the IF. This is explained by several different assumptions between both approaches: the plane strain assumption in the attenuation function of closed-form solutions in contrast to 3D attenuation in FE calculations, the presence of rigid interfaces between piles and soils in FEM, the difference in modelling the pile base reaction, the mesh influence…

Table 1. Soil model parameters used in the 3D FEM simulations.

Model / *Notation-units*	*EL*	*MC*	*HS*
Material behavior	Drained	Drained	Drained
$\gamma[kN/m^3]$	18	18	18
$E[kPa]$	25000	25000	/
ν_s [/]	0.3	0.3	0.3
c $[kPa]$	/	0	0
$\varphi'[°]$	/	33	33
$\psi[°]$	/	0	0
E_{50}^{ref} $[kPa]$	/	/	25000
$E_{oed}^{ref}[kPa]$	/	/	25000
$E_{ur}^{ref}[kPa]$	/	/	75000
m [/]	/	/	0.5

3.2.2 *Nonlinearity*

To capture the role of soil nonlinearity on the IF, the vertical displacement contours around a loaded pile are shown in Figure 8a for the case of elastic soil modelling (with an imposed displacement of 5cm=10% of d) and in the case of nonlinear soil modelling using the HS model (with an imposed settlement of 1 cm=2% of d (Figure 8b) and 5 cm=10% of d (Figure 8c).

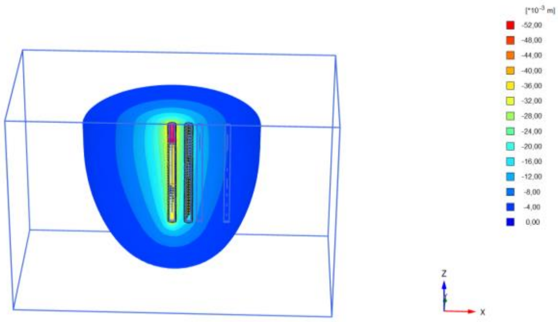
As expected, the elastic contours in the elastic analysis have a cylindrical shape and extend to more than 10 times the pile diameter. However, vertical displacement contours in nonlinear analysis have also a cylindrical shape but extends to a much smaller radius (about 3d for a displacement U=2% of d).

Table 2. Comparison of elastic IF obtained from 3D FEM simulations with those from the literature.

s/d / IFM	*3*	*5*	*10*	*Reference*
Without reinforcement effect	0.55	0.42	0.25	(Randolph and Wroth 1978)
	0.42	0.31	0.16	FE-EL
With reinforcement effect	0.45	0.34	0.21	(Mylonakis and Gazetas 1998)
	0.38	0.27	0.15	FE-EL
inhomogeneous soil	0.34	0.20	0.09	(Crispin and Leahy 2018)
	0.28	0.17	0.07	FE-EL

Consequently, the interaction between piles drastically decreases as load increases and a slippage between the 'source' pile and the soil occurs with vanishing interaction for a very high deformation (pile head displacements of about 10% of the pile diameter). It is also important to notice that even with high loading strains in nonlinear analysis, the strains (displacements) due to the 'source' pile at the interface of the "receiver" pile remain low. It is also the case for close spaced pile groups (s=3d for example). This means that the assumption of elastic soil-pile interaction in previously cited closed-form solutions remains valid and only the attenuation function (represented by the soil displacement contours in the 3D FEM calculations) should be modified. The effect of the soil model and the evolution of the IF as a function of the strain level is presented in Figure 9. It is concluded that :

- The IF decreases with the radial distance from the pile
- A good correspondence is obtained between elastic and nonlinear soil models at very low deformations.
- The IF decreases with increasing pile displacements and becomes negligible for displacements larger than 10% of d.
- With increasing pile displacement, one should be careful for the transmission of the load to the pile base. Even though the displacement due to the pile base decays more rapidly, a check of the settlement due to a load transfer to deeper soft layers is needed.



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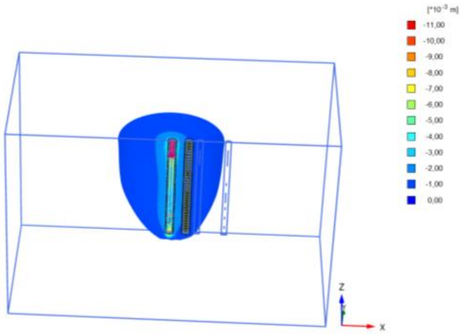

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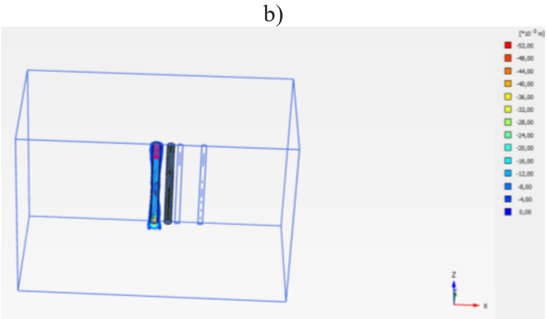


c)

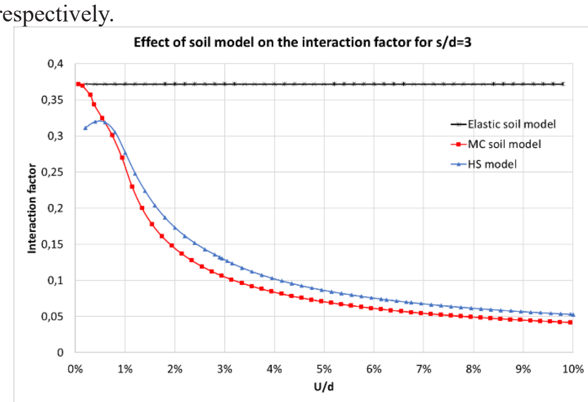
Figure 8. Vertical displacement contours in a) elastic analysis with U/d=10%, b) nonlinear analysis with U/d=2% and c) nonlinear analysis with U/d=10%.

3 PROPOSAL OF A NEW INTERACTION FACTOR METHOD

To further investigate the effect of soil nonlinearity, the interaction factors obtained from 3D simulations α_{nl} are normalized to the maximum IF at very low strains α_{elas}. Theoretically, α_{elas} represents the IF in the elastic case. Values of $\alpha_{nl}/\alpha_{elas}$ as a function of U/d and for various pile spacings s/d are given in Figure 10. Both MC and HS soil models are considered and equations of the best fit of the nonlinear IF as a function of the pile displacement are proposed. Based on these best fit equations, IF from closed-form solution could be modified to take into account the nonlinear pile-soil-pile interaction.

4 COMPARISON WITH MEASUREMENTS

A full-scale test campaign in Limelette (Belgium) reported by (Allani and Huybrechts 2019) and involving two isolated micropiles M1 and M2 and a group of 5 micropiles is considered in this paper. The tested micropiles all have a diameter of 150 mm and are installed to a depth of 6 m in a silt layer. To capture the load transfer curves, all the micropiles (isolated and grouped) were fully instrumented with Fibre Bragg Grating (FBG) optical sensors. More details are reported in (Allani and Huybrechts 2019).The proposed method using nonlinear IF is applied based on the geotechnical soil parameters for the Limelette site. Geotechnical investigations are well documented in (Maertens and Huybrechts 2003). Based on CPTs and SASW tests, two homogeneous layers are considered for the soil profile with 2 m and 4 m thickness and with shear moduli of 50 and 100 MPa, respectively.



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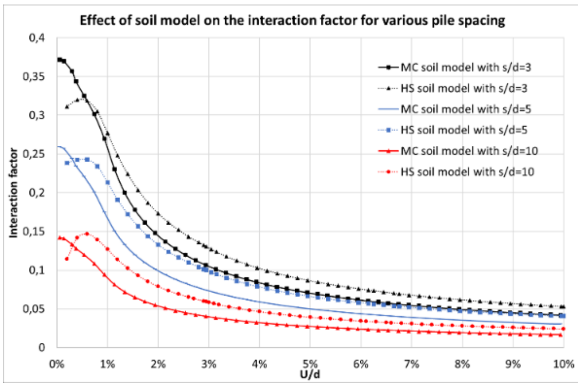
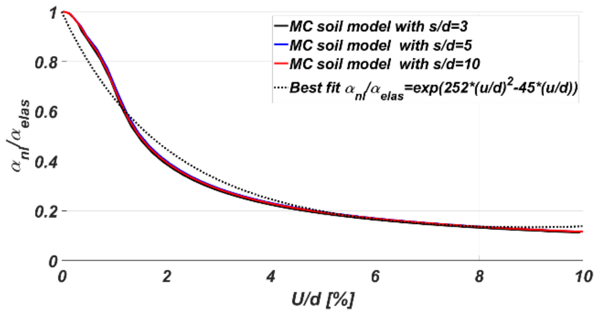
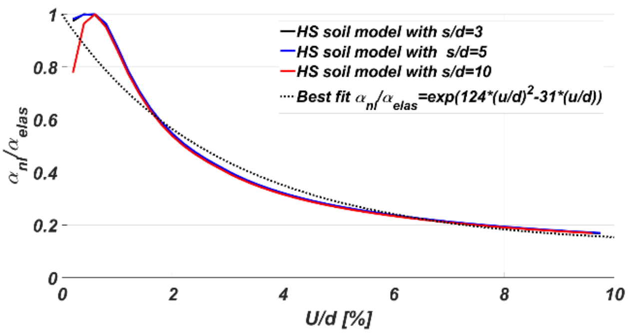


b)

Figure 9. Evolution of the IF as a function of vertical displacement a) comparison elastic analysis for s/d=3 and b) for s/d=3, 5 and 10.



a)



b)

Figure 10. Best fit of $\alpha_{nl}/\alpha_{elas}$ values for a) MC model and b) HS model.

The measured bearing load for individual micropiles is estimated to $Q_{10\%t}$=250 kN and corresponds to a pile head displacement of 10% of the micropile diameter. The initial micropile stiffness K is deduced from load-settlement curves obtained from static load tests on individual micropiles and corresponds to a value of K=50 MN/m. The nonlinear IF for neighbouring piles is calculated based on the proposed method. The IF of pile i under its own load is based on hyperbolic interpolation to obtain the ultimate load Q_{ult} . The nonlinear stiffness K_{nl} for single piles under a settlement U is expressed as :

$$K_{nl} = \frac{KQ_{ult}}{KU+Q_{ult}} \quad (6)$$

Table 3 shows the comparison between measured and calculated settlement ratio as a function of the applied average load Q_{avg} on the group of 5 micropiles. The proposed IF with both best fit equations of MC and HS soil models are used. It can be shown that:

- A good agreement between proposed IF and closed-form solutions is obtained at very low settlements
- IF from closed-form solutions is constant (independent of the load intensity)
- Group effect at very low strains is not observed in experiments
- As settlement increases (up to the working load), proposed IF decreases and more reasonable values are obtained
- Good agreement is obtained between proposed IF and experiment at the range of working loads (about half of the ultimate bearing load), a generally adopted design load for pile groups.

Table 3. Comparison between experimental results, closed-form solutions and proposed IF.

Q_{avg}[kN] / Reference	50	100	150
(Randolph & Wroth, 1978)	3.2	3.2	3.2
(Mylonakis & Gazetas, 1998)	2.0	2.0	2.0
(Mandolini & Viggiani, 1997)	2.1	2.4	2.7
(Best fit eq.in fig 10a - MC)	1.9	1.7	1.5
(Best fit eq in fig 10b. - HS)	1.9	1.9	1.7
Experiments	1.1	1.25	1.5

5 CONCLUSIONS

This study aims to address the main points of attention when applying the IF method. Moreover, it proposes a modification of the IF method by investigating the effect of soil nonlinearity with the help of 3D FEM. The main findings of this study include:

- Pile base stiffness, soil layering and soil inhomogeneity are all important aspects that may influence the pile group settlement design (up to 15-20 % according to this study).
- In contrast to conventional IF methods where the settlement group ratio is independent of the load level, 3D FEM showed a huge decrease of IF with increasing load displacement when soil nonlinearity is considered. Updated nonlinear IF values are proposed.
- A better agreement is obtained with experiments at working loads when nonlinear IF values are considered.
- At very high loading attention should be paid to the load transfer to the pile base. In that case, the IF method is no longer valid.
- Although this article shows a more nuanced picture on IF methods and proposes a practical solution to calculate modified IF taking into account the sensitivity to nonlinear soil behaviour, the assessment of pile/soil parameters remains one of the key components in foundation design.

5 ACKNOWLEDGEMENTS

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